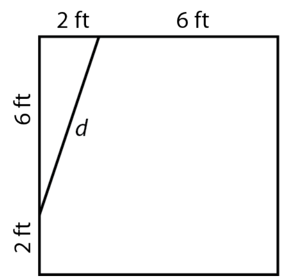
Social Distancing Squared

Teacher Stuff: Solutions with Teaching Tips

CHALLENGE 1

Solution 1 Apply the Pythagorean Theorem for both problems.



(a) $d = \sqrt{6^2 + 2^2} = 2\sqrt{10} = 6.32$ ft (1-1)

(b) $b^2 = 7^2 - (8-b)^2$ This one is purposely just a bit more challenging than the previous example. Solving this example will pave the way for tackling the more challenging examples that follow.

Rewrite the equation as $2b^2 - 16b + 15 = 0$, and solve to get $b = 4 \pm \dfrac{\sqrt{34}}{2}$. It makes sense that the two values are equidistant from the midpoint of the side because the behavior of the distance between the two neighbors is symmetric with respect to this point. (1-2)

In decimal form, the two solutions are $b = 1.08$ and 6.92 ft 6.92 and 1.08.

Teaching Tip #1

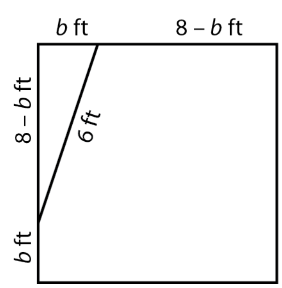
Problems Worth Solving model the exploration process that we ask students to follow in the assignments labeled as "Explorations." We start with a simple example that provides a foothold in understanding the system or phenomenon being analyzed.

Teaching Tip #2

Be on the lookout for discussion-worthy topics such as this one. You can ask questions of students when you check in with them to initial their work to see if they can explain this behavior. You can also bring it up during the class debriefing at the end.

CHALLENGE 2

Solution 3 The first two problems set the stage for figuring this one out. It's all about the diagram. We need to find the point at which the distance between the neighbors is 6 ft.



$b^2 + (8-b)^2 = 6^2$ (2-1)

Rewrite this equation as $2b^2 - 16b + 28 = 0$. (2-2)

Solve to get $b = 4 \pm \sqrt{2} = 5.414$ and 2.586 ft. (2-3)

Another way to look at this is that in order for the distance between neighbors to be greater than or equal to 6 ft, we want the locations of the people to be $\sqrt{2}$ ft away from the midpoints.

$$\text{Safe percentage} = \frac{2\left(4-\sqrt{2}\right)}{8} = 64.6\% \qquad (2\text{-}4)$$

Solution 4 The safe percentage will grow smaller with smaller squares, so the percentage will go down with 7-ft sides and up with 9-ft sides.

Solution 5 This is a repeat of the calculation made in 2-1.

$$b^2+(7-b)^2=6^2 \text{ or } 2b^2-14b+13=0 \qquad (2\text{-}5)$$

The solutions are $\frac{7}{2}\pm\frac{\sqrt{23}}{2}$. (2-6)

$$\text{The percentage is then } \frac{2\left(\frac{7}{2}-\frac{\sqrt{23}}{2}\right)}{7}=31.5\%. \qquad (2\text{-}7)$$

For $a=9$ ft, $b^2+(9-b)^2=6^2$, or $2b^2-18b+45=0$. (2-8)

The solutions are complex numbers!! Something is wrong!

What's going on here is that any point along the side is safe. The worst case scenario where neighbors are closest is when they are both at the midpoints of the sides. When neighbors are at the midpoints, their distance apart is $\frac{\sqrt{2}}{2}$ times the length of the side of the square. In this case that distance is 6.36 ft, which is greater than the 6-ft minimum.

Teaching Tip #3

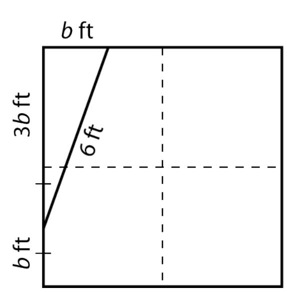
Often asking the qualitative question before asking the quantitative question will elicit more thinking because students can't go directly to some form of rote calculation. We want them to develop the habit of keeping their eyes on where they expect things to end up. This is part of evaluating solutions as they unfold.

Teaching Tip #4

We'd like to claim that we did this on purpose. Although it was an accident, we're glad that it happened. It's good for students to have to figure out the implications of getting a non-real solution.

CHALLENGE 3

Solution 7 Begin this one with a little reasoning. The same thing happens on either side of the midpoint, so we only need to consider the starting point and the point halfway to the midpoint. This creates two congruent segments—one safe and one not safe. In other words, the



safe zone on one side of the midpoint will be ¼ of the side of the square. We split the side into segments measuring b and $3b$. Now we can apply the Pythagorean Theorem.

$$b^2 + (3b)^2 = 6^2 \text{, or } 10b^2 = 36 \qquad (3\text{-}1)$$

Solve to get $b = 1.897$. The length of the side of the square is 4 times this value, or 7.59 ft.

Solution 8 The same logic as in the previous problem leads us to break the side into segments measuring b and $2b$.

$$b^2 + (2b)^2 = 6^2 \text{, or } 5b^2 = 36 \qquad (3\text{-}2)$$

Solve to get $b = 2.683$, and multiply by 3 to get a side length of 8.049 ft.

It is encouraging to notice that when the sides measured 8 ft, the percentage was 64.6%, which is just below 2/3.

Teaching Tip #5
When you initial the students' work, it's not a clerical task. It is your opportunity to make sure students understand their own solution and didn't just get an answer. It's also your opportunity to nudge them towards greater insights like this one by saying things like, "Explain to me how you can compare this to your solution to Problem 3 in order to know you're on the right track."

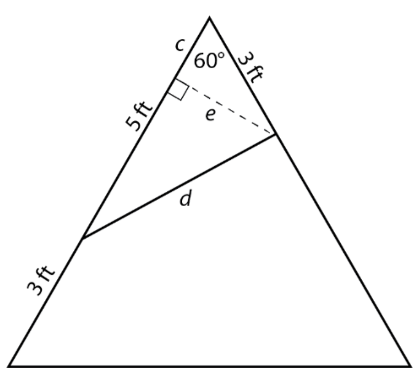
Solution 9 The smallest possible square happens when the segment connecting the midpoints (the shortest possible distance between two neighbors) is 6 ft. Students can approach this in a variety of ways based on what they have learned so far. We'll use special triangles.

$$\frac{\sqrt{2}}{2}a = 6, \text{ so } a = 6\sqrt{2} = 8.485 \text{ ft} \qquad (3\text{-}3)$$

Once again, it's encouraging to see that we are on the right track by remembering that the 9-ft square was safe across the entire side.

Challenge 4

Solution 11 There are many good options. We're going to use special triangles to find the labeled sides.

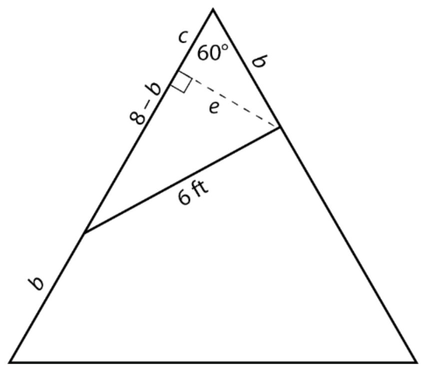


$c = \dfrac{3}{2}$ ft, and $e = \dfrac{3\sqrt{3}}{2}$ ft.

Now we can apply the Pythagorean Theorem.

$$d=\sqrt{\left(5-\frac{3}{2}\right)^2+\left(\frac{3\sqrt{3}}{2}\right)^2}=\sqrt{\frac{49}{4}+\frac{27}{4}}=\frac{1}{2}\sqrt{76}=4.36\text{ ft} \qquad (4\text{-}1)$$

Solution 12 We provided the previous problem so that it would pave the way for this one.



$c=\frac{b}{2}$ ft and $e=\frac{b\sqrt{3}}{2}$ ft

Apply the Pythagorean Theorem.

$$\left(8-b-\frac{b}{2}\right)^2+\left(\frac{b\sqrt{3}}{2}\right)^2=6^2 \qquad (4\text{-}2)$$

This simplifies to $3b^2-24b+28=0$. (4-3)

As was the case with the square, the two solutions are equidistant from the midpoint of the side.

$$b=4\pm\frac{2\sqrt{15}}{3}\text{ ft} \qquad (4\text{-}4)$$

The safe percentage = $\dfrac{2\left(4-\dfrac{2\sqrt{15}}{3}\right)}{8}=35.4\%$. (4-5)

Teaching Tip #6

When you are aware of the nuances and complexities of the problem students will eventually solve, one way to help them without helping them is to provide simpler versions that will get them ready for the more difficult versions. This is one of many techniques we recommend for guiding students in the problem solving process.

CHALLENGE 5

Solution 14 We've introduced a new element. Students have to dig into the archives to remember how to find the constant for an inverse relationship. This this one is an inverse of the square, but they should see that it works the same way. We provided two sample values on the graph. Pick either one of them to find the constant of variation.

Teaching Tip #7

We do this a lot in this course. We like to change things up so that students always think about the tools and techniques required rather than follow routine solution paths. That's what this course is about!

Let the danger index = $\frac{k}{d^2}$, where d is the distance between neighbors. Substitute values: $1 = \frac{k}{6^2}$, so $k = 36$. We can use our equation, danger index $= \frac{36}{d^2}$, to find the distance that gives us a danger index of 2.

$$2 = \frac{36}{d^2}, \text{ so } d = 4.2426 \text{ ft} \qquad (5\text{-}1)$$

Now the problem is like Problem 3 but with a distance of 4.2426 ft instead of a distance of 6 ft.

$$b^2 + (6-b)^2 = 4.2426^2 \qquad (5\text{-}2)$$

Solve to get $b = 3$ ft. Once again, we have given the students values that lead to an unusual and unexpected solution, and that will require interpretation.

In this case, $b = 3$ ft tells us that neighbors will be 4.24 ft apart (with a danger index of 2) when they are at the midpoints of the 6-ft sides. In other words, everyone will be safe 100% anywhere along the sides of the square. During the debriefing, have students solve the problem with a desired danger index of 1.5 instead of 2. (This will lead to 42.3% of the side of a 6-ft square being safe.) Read "An Extra Helping" on the student assignment and Teaching Tip #8 to the right. There is a lot more you can do!

Teaching Tip #8

You have SO much more you can do with the danger index. The only reason we didn't do more is that we ran out of room. There are all kinds of reversals and variations you can explore with your students. Does the model we used make sense? What about an inverse relationship or a linear relationship? What if we designed the square to yield a specific danger index? What if we wanted to know that average danger index across the side of a square or triangle? What about other shapes? We'll leave further exploration up to you. There's plenty of uncharted territory if you're interested.