Social Distancing Squared

Use a variety of math techniques to analyze the safe zones on a bench.

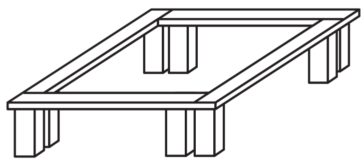

Figure 1: Social Distancing Bench

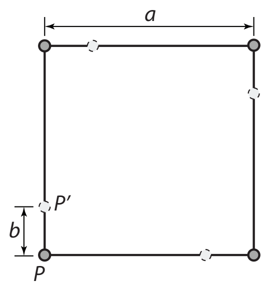

Figure 2: Square of Four

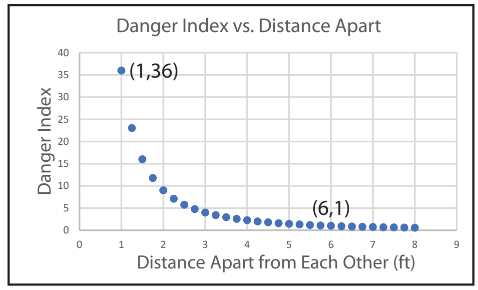

Figure 3: Danger Index

Introduction

Math is everywhere once you practice seeing it. Sometimes it shows up to entertain you when you least expect it.

Recently I went with members of my household to a beautiful farm wedding venue that had been converted into a socially safe gathering spot during the pandemic. The farm had an expansive patio on top of a hill overlooking mountains, valleys, and a wonderful sunset. I sat together with my family on a set of benches like the ones shown in Figure 1, where we enjoyed food, beverages, and conversation. The bench squares were about 20 ft apart from each other, so the arrangement felt safe. It was nice to have a chance to get out of the house and enjoy each other's company in such a beautiful outdoor spot.

Then math arrived, which to me always adds another lens through which to enjoy my surroundings. The members of my group happened to be from the same household, so we were used to being together, but I began wondering how a group of four strangers would use the benches. If the square were 6 ft on a side, then the strangers could sit on the four corners of the square and be safe. If all four moved a distance b, as shown in Figure 2, they would no longer be within the minimum distance of 6 ft apart. If the square were 8 ft on a side, there would be some places where the four people could be safe and other places where they would not be safe. I'm going to lead you through an exploration of the mathematics of all of this. There are plenty of interesting relationships to be found, and maybe you'll find some other relationships worth exploring on your own.

An Extra Helping

- ✓ Return to Challenge 2, but instead of giving the side of the square a value, let it be represented by a. Derive a function $P(a)$ that shows how the percentage of the side along which the people are safe depends on the length of the square's side.
- ✓ Ask what-if questions and challenge assumptions in order to generate new paths of exploration related to this topic.
- ✓ Try other regular polygons. A hexagon would be a good place to start.

Just for Fun

- ✓ Describe a time where right in the middle of doing something else you thought of how math or science would relate to your situation.
- ✓ Design and draw a protective face mask that will amuse or inform people or that will reveal something about you that most people don't generally know.

Challenge 1

When we explore and analyze a phenomenon using math, we begin with the simplest example so that we can wrap our heads around the basic idea.

1. In Figure 2, *a* represents the side of a square, and *b* represents the distance that each person has moved. (a) If $a = 8$ ft and $b = 2$ ft, what is the distance between neighbors after they have moved? (b) Suppose everyone moves so that the distance between neighbors is 7 ft. How far is everyone from their nearest corner?
2. Have the teacher initial your work.

Challenge 2

Now we can analyze the safety of the square in a broader sense. As the four people move around the perimeter of the 8-ft square, some places they are safe because they are at a minimum distance apart of 6 ft, and some places they are not safe.

3. As the people move along the sides of the square from one vertex to the next, along what percent of the distance are they safe?
4. If we make the square 7 ft on a side, will it increase or decrease the percentage of the distance along which the people are safe? How about if we make the square 9 ft on a side?
5. For what percent of the side of the square are people safe using the side dimensions in the previous problem?
6. Have the teacher initial your work.

Challenge 3

One way to explore further is to reverse the direction of our thinking. In other words, instead of working towards the percentage of distance along which the people are safe, we'll start with that percentage and work backwards.

7. Suppose you want the people to maintain their 6-ft minimum distances, but you want them to be safe along exactly 50% of the side of the square. How big should the square be?
8. How big should the square be if you want the people to be safe along 2/3 of the side?
9. What is the minimum square size that will allow the people to be safe along 100% of the length of the side?
10. Have the teacher initial your work.

Challenge 4

Now what? Start asking what-if questions. For instance, what if the benches were not a square?

11. As before, we'll examine the simplest example we can think of related to this new path. The benches are in the shape of an equilateral triangle with sides that measure 8 ft. If the three people on the vertices each move 3 ft along the sides in a clockwise direction, how far apart will they be from each other?
12. In this same triangle, along what percent of the side will the people be safe by maintaining a 6 ft or greater distance apart from each other?
13. Have the teacher initial your work.

Challenge 5

We could explore other polygons, but instead we're going to explore a new direction by complicating one of our assumptions. So far we have considered distances to be safe or unsafe. That's probably not entirely accurate. Certainly being 1 ft apart is less safe than being 6 ft apart, right?

14. We're guessing that the danger of being near someone without a mask varies inversely with the square of the distance they are apart. (We don't <u>know</u> that this model is accurate, but we are thinking that it might behave similarly to light intensity, which varies inversely with the square of the distance from the light source.) In the 6-ft square in the introduction, the people could not move off the vertices of the square if they wanted to be safe. What if the danger index were allowed to get up to 2? Use the data in Figure 3 to find the percentage of the side of a 6-ft square along which the danger index will be less than or equal to 2.