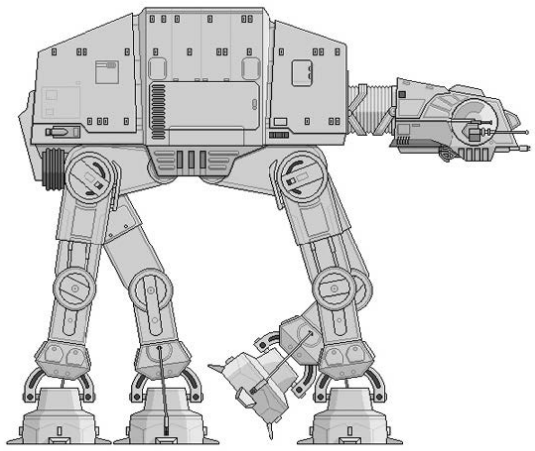
The Empire Stumbles Back

Project

Bad Science

If you encountered an AT-AT while on a family ski vacation, you might have second thoughts about buying that time share on the icy planet of Hoth. Blasts from a laser cannon make moguls look like a walk on the beach. Nevertheless, this formidable quadruped has a tragic engineering flaw that should make you feel less afraid. For starters, any vehicle that can be brought down by tripping it with wire should never make it off the drawing board. In this investigation, you'll use a variety of math skills to analyze the vehicle's even more tragic flaw.



Thinking About Tipping

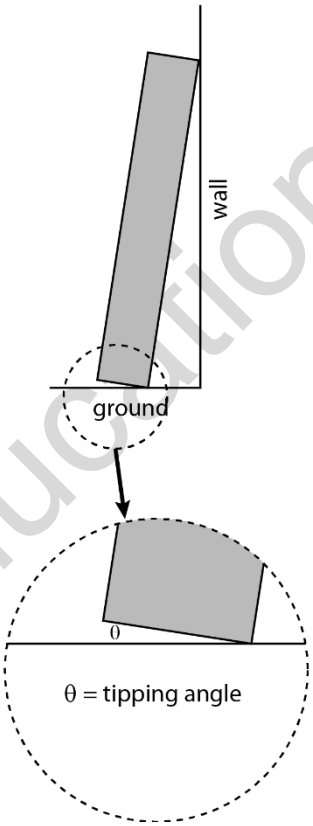
1. Stand your 2x4 on end. From this position, it can tip along one of the 3-1/2" edges or along one of the 1-1/2" edges. Does it tip one way easier than the other way? If so, why do you think this happens? (Yes, these are the real dimensions of 2x4 lumber.)
2. For the first experiment, you'll be tipping the lumber along the 1-1/2" edge. What are the attributes of the 2x4 that will affect the angle at which it is just barely balanced? (We'll call this the "tipping point.")
3. There's a point on the 2x4 that's VERY important when it comes to tipping. Try to notice what's going on with the geometry of the 2x4 (and with this important point if you know what it is) when it's right at its tipping point.
4. Have the teacher initial your work.

Calculate & Predict

5. Use your observations from the previous section to calculate the tipping angle (shown on the next page) of a 16.5-in 2x4 when tipped on the 1-1/2" edge.
6. Now use your technique to calculate the tipping angle of <u>your</u> 2x4 along its 1-1/2" edge and then along its 3-1/2" edge.
7. Have the teacher check your predictions and initial your work.

Experiment & Measure

8. Develop a technique for measuring the tipping angle. You can lean the board against the wall (or other vertical surface) until it's at its tipping angle. Make two measurements, and then calculate the angle.
9. Use this measurement technique for both directions of tipping. Compare these two values with the two values you predicted in the previous section. Calculate the percent error for each measurement.
10. If your errors were both under 10%, go on to the next section. If not, go back and repeat calculations and/or measurements until you can reduce the errors below 10%.
11. Have the teacher initial your work.



Develop a Formula

12. Let a the length of the 2x4, and let b = the width of the edge along which the 2x4 is being tipped. Develop a formula that show how θ depends on a, b.
13. Make sure your formula works with the answers from the previous sections.
14. Have the teacher initial your work.

Putting Your Formula to Work

15. What length would the 2x4 have to be in order for it to have a tipping angle of 15° along the 1-1/2" edge?
16. What width would a 20-in.-long board need in order to have a tipping angle of 18°?
17. Determine the cross-sectional dimensions of a 15-in.-long board that has a tipping angle of 8° in one direction and 12° in the perpendicular direction.
18. Sketch a combination of two 2x4 pieces glued together that will tip at one angle in one direction but a completely different angle in the opposite direction. Be prepared to explain to the teacher what would make the tipping angles different.

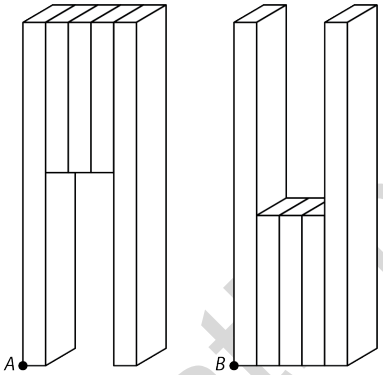
Intermission

If you're continuing to the more interesting but difficult challenges (and we hope you are), read and study "AG.04.LS-01." This is a Learning Snack that the teacher will provide.

The Empire vs. the NFL

19. Make sure you know how to find the center of mass of a composite shape. If you don't, go back and read the document that was given to you on this topic during the intermission.

20. The two composite shapes to the right are made from the same amount of wood. The long 2x4 pieces are 20 in. long, and the short ones are 8 in. long. The left is a model of an AT-AT from "The Empire Strikes Back." The one on the right is a model of an NFL guard…with antlers. Which one will have a higher tipping angle when tipped along the 3-1/2" edge of the outer 2x4? Explain your choice.



Empire's AT-AT NFL Guard

21. Have the teacher initial your work.

22. Use what you learned about the center of mass to locate the coordinates of the center of mass of the composite shape in each orientation. Give the coordinates relative to points *A* and *B*.

23. Once you know where the center of mass is, calculate the tipping angle for both orientations of the composite shape.

24. What percent greater is the larger tipping angle compared to the smaller tipping angle?

25. Have the teacher initial your work.

More Tipping (Follow the teacher's directions.)

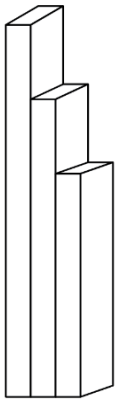
26. The teacher may have provided (or may have had you provide) composite shapes that you can test. For each one, calculate the tipping angle, and then use the lean-against-the-wall method to test your prediction.

Are You Ready for These?

27. The composite shape to the right will tip at different angles to the left and right. The lengths of the 2x4s are 40 in., 32 in., and 24 in. Find the tipping angle both to the left and right.

28. Design a composite shape from two 2x4s that tips at 12° to the left and 15° to the right. (You might find the tool you develop in the next challenge to be useful.)

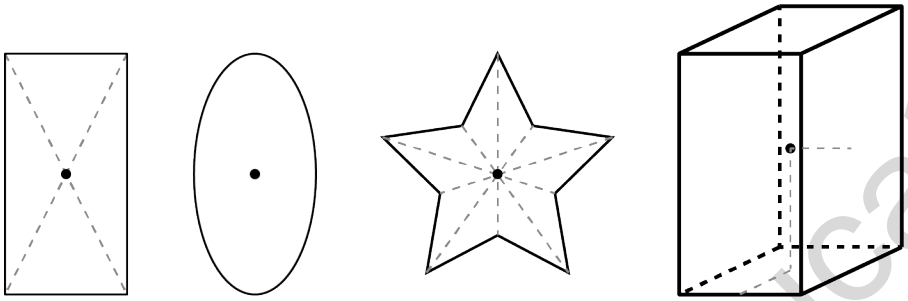
29. You're several units into this course and have been doing Think-Like-a-Programmer challenges that invite you to develop spreadsheets or programs for handling routine calculations. Once you understand the method of finding the center of mass and calculating tipping angles, everything else is fairly routine and lends itself well to an automated process. Create a spreadsheet or program to solve problems like those in this section and the previous two sections.


Assymetrical Tipper

Center of Mass

Learning Snack

Common Sense

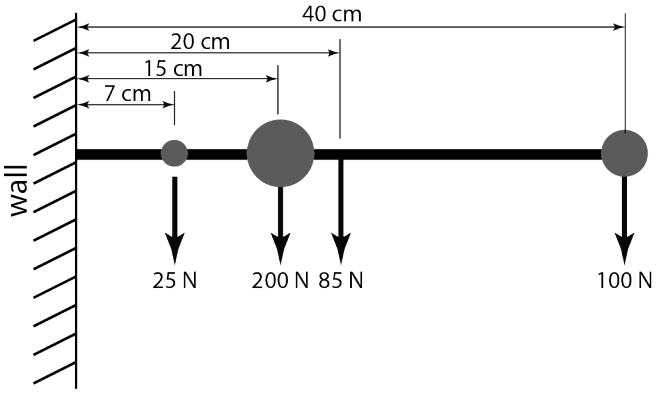


For shapes with symmetry, we have an intuitive sense of where the center of mass is located. It's in the center, right? This is great for party conversation, but it's not scientific.

Conveniently Located

The center of mass makes analyzing the motion of an object much simpler. Suppose you threw the block on the right and wanted to use physics equations to figure out where it would land. A complicated way of analyzing the path of the block would be to look at what happens to each tiny particle that makes up the block. Not fun. Fortunately, we can imagine all the mass of the block concentrated at the center of mass, which allows us to write equations for a single point.

A Simplified Model



There are three spheres mounted on a rod that is attached to the wall. Both the spheres and the rod have mass, so gravity pulls them downward. This is indicated by the arrows representing the forces. Each force is trying to twist the rod clockwise around the point where it is attached to the wall. This twisting is called "torque" or "moment." The moment contributed by one force is given by this equation: $\text{moment} = \text{force} \cdot \text{distance}$. If you have ever studied levers, you know that this makes sense. We can increase the moment by increasing either the force we apply or the distance from the fulcrum, which in this case is the joint at the wall.

1. Find the total moment of the four forces on the rod and spheres.
2. Have the teacher initial your work.

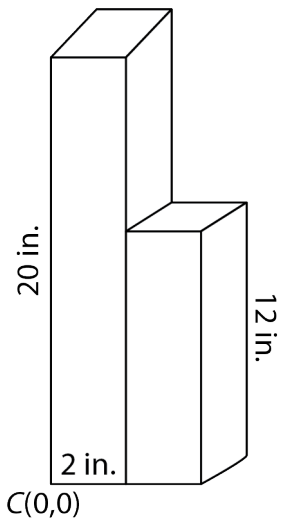
3. Imagine taking all four forces and concentrating them at a single point on the rod that is a distance, *x*, from the wall. This point is called the "center of gravity" or "center of mass."
4. Find *x*.
5. There, you've done it; you've found your first center of mass mathematically.

Develop a Formula

6. Since there is a direct variation between the force contributed by each object and its mass, we could have used mass in our calculations instead of force. Make sure you understand this before going on.
7. Suppose you have three masses on a rod represented by m_1, m_2, and m_3. The distances that these masses are from the wall are d_1, d_2, and d_3. The rod has a mass of m_4 and its center is a distance, d_4, from the wall. Let *x* = the distance from the wall to the center of mass. Write a formula for calculating *x* from the other eight variables.
8. (Optional) How is this formula similar to the formula that is sometimes used to calculate weighted averages when determining your grade.
9. Have the teacher initial your work.

Apply Your Formula

The diagram shows two blocks of woods that have been glued together. Each has a cross-section of 3 in. x 2 in. When finding the center of mass of this composite shape, you'll have to use your formula from the previous step once in the *x*-direction and then again in the *y*-direction.



10. Explain why you don't really need to know anything about the density of the wood. What will you use instead of mass in your equations?
11. Calculate the coordinates of the center of mass of this composite shape.
12. Have the teacher initial your work.

