Geometry Teacher's Pets

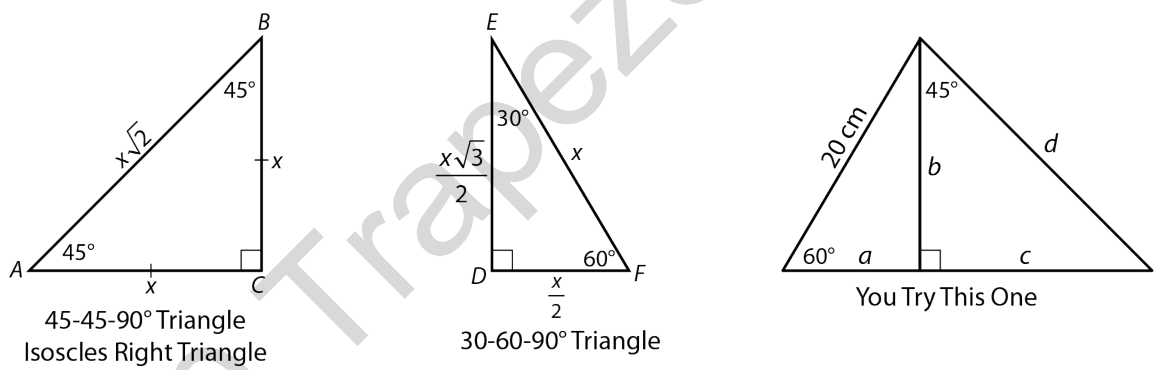
How Does THAT Work?

Special Triangles

- The special triangles get to choose their donuts before the rest of the class.
- The teacher always believes the special triangles' excuses for not having their homework done.
- The special triangles sometimes get to work at the teacher's desk next to the radiator.
- The special triangles have their own private helicopter that takes them to and from school.
- What privileges do YOU think the special triangles get?

Trigonometry Without a Calculator?

You've been using the sine, cosine, and tangent functions to find missing sides in right triangles. It turns out that there are two triangles for which you can find missing sides without a calculator. These two triangles are the special triangles. Memorizing these ratios is well worth your while because they come up a lot in your next few courses, and you will undoubtedly see them on SAT questions. Here are the two diagrams you need:



To make sure you understand, write the values of the four variables in the right-hand diagram. You may not use your calculator, and you should leave your answers in simplified radical form.

Have the teacher initial your work: __________

Not So Fast!

Did you think the teacher was going to give you a free Mustang convertible for memorizing the ratios? The teacher is far too concerned about the development of your mind to do that. Use what you know about the Pythagorean Theorem and simplifying radicals to derive these ratios. Try the 45-45-90° triangle first. Once you've done that, try the 30-60-90 triangle. This one is a tiny bit more difficult, so we'll give you one hint: equilateral triangles.

When you think you're ready to explain, have the teacher initial your work: __________

Great work so far. We're going to share with you a few tips that will help you work with these ratios in other challenges the teacher will give you in this unit and in future units.

30-60-90˚ Another Way

The ratios shown on the previous page are one way to look at the 30-60-90° triangle. Notice that in the diagram to the right, we've assigned x to $\overline{DF}$ instead of to the hypotenuse. Find DE and EF in terms of x. Looking at the ratios in this way will often come in handy.

45-45-90˚ in Reverse

Suppose the hypotenuse of an isosceles right triangle is 5 cm. What would you have to divide this by to find the length of the legs? Write it. Now rewrite this ratio so that there is no radical in the denominator.

Repeat what you just did, but this time use a measure of 7 cm for the hypotenuse. Write the measure of the leg without a radical in the denominator. Are you noticing a pattern?

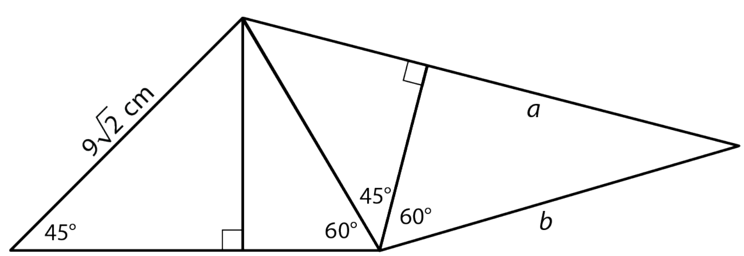
Repeat the procedure one more time using a measure of 11 cm for the hypotenuse. Try to get to the final simplified answer in a single step by the following the pattern you have been noticing.

Complete the following sentence: "Dividing by $\sqrt{2}$ is the same as multiplying by ________.

Have the teacher initial your work: __________

Your Final Test

Without using a calculator, find a and b. Leave your answers in simplified-radical form.



What's in a Name?

How Does THAT Work?

Centroid

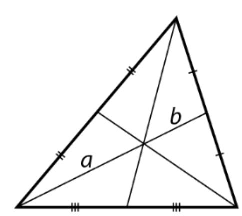
- The year is 2120. Robots have been given the right to vote. (They also make great French toast and cannoli.) Like their human counterparts, robots don't always see eye unit-to-eye unit when it comes to politics. Although they are as politically polarized as people, some robots remain in the middle. A *centroid* is a robot that leans neither left nor right and tends to support candidates based on their policies rather than on their party.
- *Centroid* is a kid on my basketball team. His parents are eccentric. He has a brother named Vertex and a sister named Scalene. His dog's name is Steve. He prefers vanilla ice cream.
- A *centroid* is a type of blister that can form on the hands or arms of someone who has made contact with a space rock. The best-known treatment is to apply—of all things—peanut butter. Chunky works best.
- A *centroid* is half human and half centipede. You do not want to find one of these in your bedroom closet. They tend to borrow clothes without asking.
- What do YOU think a *centroid* is? We're looking for fun—not accurate.

Will the Real Centroid, Please Stand Up?

A centroid is the intersection of the three medians of a triangle. (Although you should already know what a median is, we'll remind you that a median is a segment that connects a vertex of a triangle to the midpoint of the opposite side.) The centroid has some interesting properties that you'll discover.

Seriously. You've Got to be Kidding

Sometimes in this chaotic complex universe in which we hang out, we come across a bit of mathematical simplicity that makes us stop and say, "What?" Sometimes we're so dumbfounded that we're unable to form consonants. The centroid is one of those things that causes this reaction. It just happens to lie at a point on the median that splits the median into two segments where one is EXACTLY twice the length of the other. Yup, it's true. In the diagram, $a = 2b$.



Your Challenge

Prove it. Show why $a = 2b$. Don't worry about the formal proof for now. Just figure out how it works. By the way, in the Coordinate-Geometry unit you figured out that a midsegment was parallel to and half the length of the side it does not touch. Do you remember that? We just thought we'd mention it. Be ready to explain your technique to the teacher.

Have the teacher initial your work: __________

Formal Proof

Take your work from the previous section and put it in the form of a flow proof. Often it's helpful to think through why something works the way it does before diving into the formal description.

Have the teacher initial your work: __________

But Wait, There's More

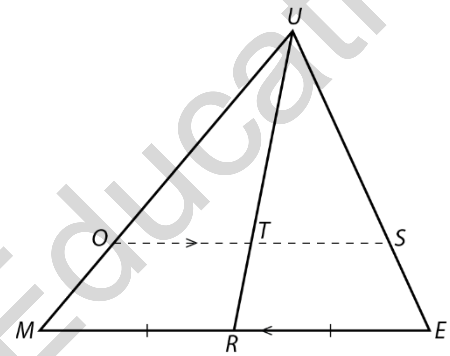
We're going to reveal another cool fact about the centroid, but before we do, prove the following.

INTERMISSION

Given: $\overline{UR}$ is a median and $\overline{OS} \parallel \overline{ME}$.

Prove: T is the midpoint of $\overline{OS}$. (Use a paragraph proof.)

Comments: $\overline{OS}$ is drawn at a random location along the median. You're proving that the midpoint of a randomly-drawn parallel always lies on the median. This will be useful soon.



Have the teacher initial your work: __________

The Cool Thing

If you have not completed the previous section, go back. You must earn the right to read this cool thing. It's written in code, but we're pretty sure you can figure out how to break the code.

> The centroid of a triangle is also the center of mass of the triangle. If you made a triangle out of wood, it would balance right on the centroid. Crazy, right?

Now prove it. Why is this true? You don't have to write a formal proof. It will be fine to come up with a logical explanation that you can share with the teacher. We're going to give you one word that may or may not help you. The word is "slices."

Have the teacher initial your work: __________

The Grand Finale

Yes, that's right there's even more! Seeing the cool things associated with the centroid is as exciting as seeing a shooting star…one that draws a picture of dancing camels in the sky.

Look at the drawing on the first page. Would you believe us if we told you that the medians divide the triangle into six triangles that all have the same area? We wouldn't believe us either, and we'd probably call the authorities. In fact, it's true. Prove it. As in the pervious section, it will suffice to come up with diagrams, equations, and logical reasoning that you can share with the teacher.