VELOCITY & SIMILARITY

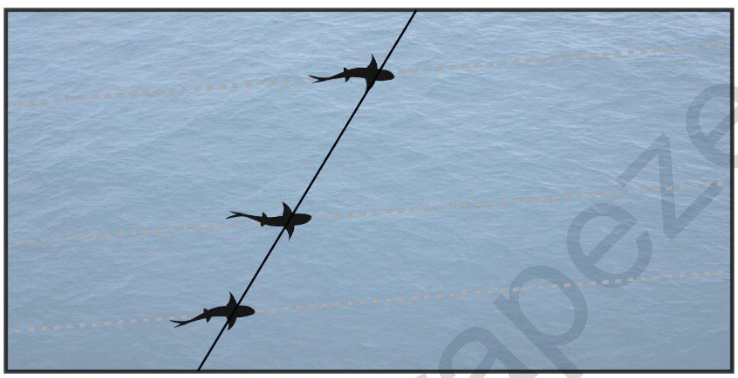
The Challenge Zones

Since you're new to explorations, we're going to do a few things to help you. First, we'll give you the question on which your exploration will be based:

> "I know all about similar triangles and distances. Will similarity and proportions also apply in problems involving velocity? Hmmm…I wonder."

Second, we have provided you with the diagrams you need for two different scenarios related to the question. In the future, this is all your responsibility. Your job now is to explore the situations described and to develop the math needed to address the question asked.

Synchronized Swimmers

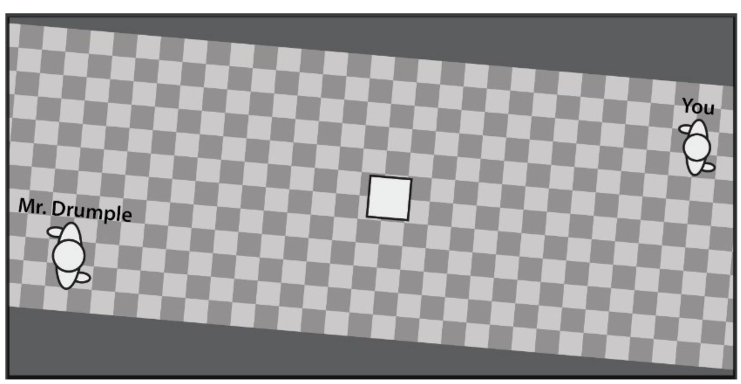


These sharks are preparing for the Shark Olympics. One of their tricks is remain collinear at all times as they swim along parallel lines. Unfortunately, they tend to swim at different speeds. Explore the math that relates to this problem, and make recommendations for the sharks.

One of the habits you'll develop as you explore this year is to see if the math developed for one application can be used for another application. Explore the following to see if it is at all related to the synchronized sharks.

Dodging Drumple

You are now three days late turning in your Civil-War essay. Mr. Drumple is not happy and you know it. You notice him walking down the hallway towards you, but fortunately there is a post that blocks his view of you. If you don't walk at just the right speed, he may up seeing you. Develop the math to keep yourself safe.



Explorer's Guide

Finding Your Own Territory

Dear Students:

It's likely that most of your math experience has involved applying mathematics in ways that have been demonstrated for you. We wish more for you than this. We want you to discover ways to apply mathematics on your own without needing someone to show you how. We want you not only to solve problems but also to identify the problems that need to be solved. You can practice by walking down well-worn paths, but our bigger goal for you is that you will be forging paths of your own. Isn't that what explorers do?

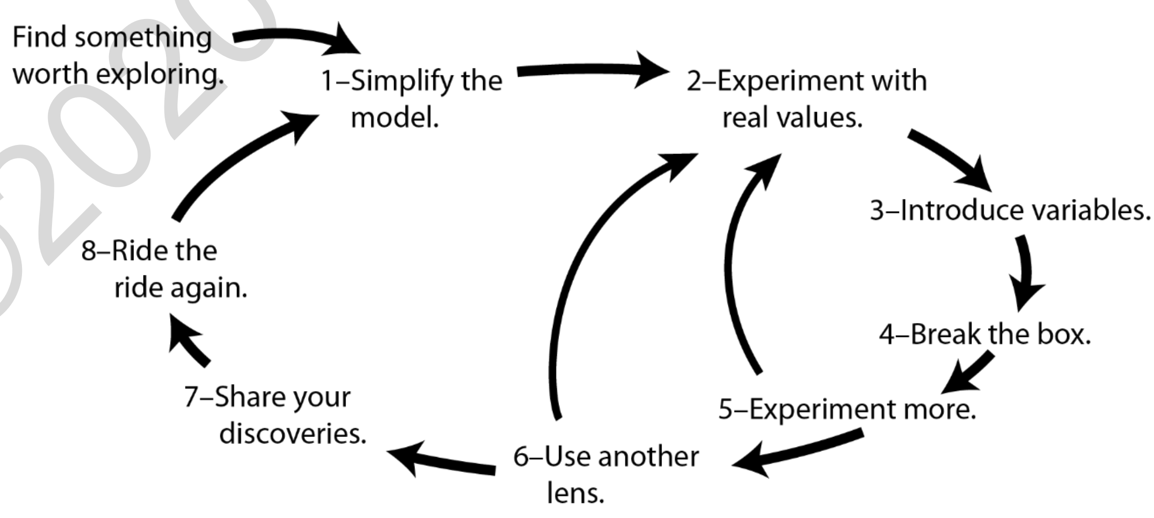
Sincerely,

Math Teachers of the World

The Exploration Process

Here are three important things you should know about the exploration process:

1. Exploration is not linear: it's not like knitting a sweater or building a model ship. There is no pattern to follow or directions that you can go through from beginning to end. As the diagram below shows, exploration will require continually cycling back to earlier steps in the process. Also, you're never finished: there's always more exploring to be done.
2. The ability to explore (just like the ability to create) is not something you are either born with or not. You can and will develop your ability to explore with practice.
3. Exploration is difficult. We mention this because you may not be used to exploring without step-by-step directions, so to some of you it might be uncomfortable. Trust that you will get more comfortable with practice. Exploration is also fun. Remember building with blocks when you were little…or last week? When you explore in mathematics you can create things that are uniquely yours, just as you do with blocks.



Find Something Worth Exploring

The process starts with identifying a system or phenomenon that could be analyzed with mathematics. Ask yourself questions like one of these examples.

- **Example 1:** I wonder if there is way to calculate the length of white segments on the highway so that they appear to be a specified length from the angled viewpoint of the driver.
- **Example 2:** How can I use math (and maybe a spreadsheet) to track the path of a billiard ball as it bounces off the four sides of the table?

Step 1: Simplify the Model

It's important to start with a problem that is a manageable size. Simplify the problem to its most basic form so you can learn about it before considering more-complex variations. Draw a diagram and identify the dimensions and other quantities with which you'll experiment. These will be your variable quantities.

Also decide which variables you're <u>not</u> going to consider. You could choose to do this for two reasons: (1) You want to omit the variable to keep the model simple, in which case you should set it to a reasonable constant. (2) You don't view the variable as having an impact on the solution…at least not to the level of accuracy in which you're interested. In this case, the variable does not need to have a value. (Remember that later in the process when your model becomes more complex, you can change your mind about any of the decisions you make now.)

Step 2: Experiment with Real Values

Begin playing with actual values in your simplified model. Set everything but one variable to a constant, and see if you can calculate the value of that variable. Reverse it. Turn it inside out. If there are five different variables you are considering, try setting up a problem to solve for each. Working with your simplified model in this way will help you discover the math topics and procedures that apply. These will still apply when you start looking at the problem more generically in the next step.

Step 3: Introduce Variables

Once you understand how mathematics is applied to your model, you can use multiple variables. Follow the same calculation steps you followed when values were real. Try to solve for each of the variables. Here are some other things to look for:

- In general, how does each variable affect the others? What relationships can I find between variables? Linear? Quadratic? Direct or indirect variation?
- If there is a relationship I don't recognize, can I put an example problem into a spreadsheet or graphing calculator to create a visual representation of that relationship?
- Was there an interesting pattern? A maximum or minimum? A variable that dropped out?

Step 4: Break the Box

Now that you've got the general idea about what's going on in this problem, you can begin to break out of the simple box. Identify the constraints you put on the problem and change them. Ask lots of what-if questions to generate new variations.

Prioritize your list of variations to explore. You want to start with the ones that will be closest to the original in order not to build too much complexity too fast. You also want to consider which variations might be more intriguing or yield interesting or useful results.

Step 5: Experiment More

Now that you have more variations, you need to explore them just like you did the simplified model. Go back to Steps 2 and 3. This will lead to new discoveries. By the time you end up back at this step, you might also have thought of new variations. If so, you know where to go. Complete this loop until you run out of possibilities.

Step 6: Use Another Lens

Maybe you missed something back in Step 2. Is there a completely different mathematical approach that could have been applied? If so, it might help you understand better. It might even reveal new insights. If you even suspect that another approach might be worthwhile, you should return to Step 2 to try it.

Step 7: Share Your Discoveries

There are two things left to do. First, reflect on what you discovered. What was interesting? What was revealed? What was learned? What are the implications of the discovery? Second, think about what is worth sharing and how you might share it.

Here are some ideas for the types of things you can communicate to or share with others. As you get better at exploring, you might come up with some creative ideas of your own that you can add to this list.

1. What are the discoveries that are most revealing, interesting, or surprising?
2. Did you find a really cool application of a specific theorem or math technique? How about formulas that were surprisingly simple and elegant?
3. Sometimes graphs reveal the behavior of a system more clearly than equations. Are there relationships that are worth graphing?
4. Is there a diagram or model that you could create that would show something you discovered?
5. Could you further process data with a spreadsheet or computer program? Would a simulation reveal new discoveries?

6. Do you have a way to test predictions? Do it. What's more exciting than finding out that you can create a math model that actually works? Having that power makes you a superhero, right?

Step 8: Ride the Ride Again

Discovery leads to discovery. The more you explore and model systems and phenomena with mathematics, the more you'll see how a model applied to one situation could also be applied to another situation in a completely different discipline. For instance, the same math that describes how your investment in a mutual fund grows can be used to describe how a microorganism population grows in a culture.

You should be on the lookout for these types of connections. If you find one worth analyzing—and you have the time—bring it into Step 1. If not, you now have one more model and set of techniques that you can use to analyze and understand other systems and phenomena. Be on the lookout for them.

