A Problem Solving Classroom

by Mark Illingworth

Sign in and put on your security tag. I'm going to bring you into a classroom that has embraced both the goals and materials of this course. If you're looking to become a problem solving teacher, it will be helpful to see what this type of classroom might look like halfway through the year after the students have had a chance to embrace a new level of challenge. It's important for you to know that the class we're visiting also works on Core Concepts (as you would hope), but we're choosing to visit the class during a problem solving session. You should also know that any named characters in this narrative represent real students' interactions with me, although admittedly many of these interactions have been compressed into a single class period.

1. The class is already in session when you enter. They're working in teams of three on a Problems Worth Solving called "Vortex," which has to do with finding patterns in spirals made from nested similar polygons. (The assignment is "AG.05.PWS-04 Vortex.")

2. Listening to one of the groups, you notice how well one student articulates his idea for figuring out the next step in a problem. You also notice how well the other two students pay attention. One of them challenges the speaker's assertions. She makes a diagram to illustrate her point, and the three of them talk to resolve the discrepancy. You can tell that students have become accustomed to discussing mathematical ideas.

3. You see that one of the groups has a red cup on their table. All the other tables have blue cups. You go over and ask the students what the red cup means. They explain that this is how they let the teacher know that they're stuck, and they show you a few things they've tried unsuccessfully. You're impressed both by how long they've been trying and by how many ideas they were able to test. The teacher shows up, and they explain again what they've done so far. The teacher asks them what they usually do when they have quantities that are unknown. "Use a variable," one student replies. "Well, have you done that with any of the segments?" the teacher asks. There is a pause. "Once you do that, you can start finding expressions for other sides, right?" You can see that the students know that this is all they will get for now from the teacher, and they set about creating a new diagram… this one with the shortest side of a triangle labeled x. The teacher moves to another red cup that has popped up, and you follow.

4. This group is not stuck. They have apparently completed all the questions in Challenge 2, and they are needing the teacher's initials as requested in the assignment. The teacher asks them to explain what they can learn from the graphs on their calculators. She also asks them to explain how things would have turned out differently if they had been working with pentagons rather

than squares. Together the students answer both questions, and the teacher initials their papers. The taller of the students says, “My turn!” and she goes to a table on the side of the class and takes a yellow marble from a bowl and drops it from about 3 ft into a jar. You can hear the clack of glass on glass. The other group members explain that this is how the class records successes. You ask if yellow is their group’s color, and they explain that they don’t have colors and that the whole class works together. When the jar is full, they’ll get to plan a class event.

5. When the teacher finishes working with the group, she turns and Keely is standing behind her and has something she wants to share. They go to an unused desk, and Keely sits down and opens up a spiral-bound notebook to a page where she has drawn a person made up of rectangular prisms. She tells the teacher that she took her suggestion and researched angular momentum, and she plans to model a dancer with rectangular prisms so that she can calculate their moments of inertia. The teacher suggests to Keely that she make some initial calculations to make sure that she’ll actually be able to follow through on this idea. Later, she explains to me that Keely is a dancer, and she wanted to do an end-of-year project related to dancing. Even though the project culminates in May, the students start formulating their ideas in the winter. Sometimes they need small hints from the teacher to find ideas worth exploring. The teacher had suggested to Keely that she research angular momentum to see how it might relate to how a dancer spins faster when she pulls her arms in. From there it will be up to Keely.

6. When Keely returns to her group, the teacher shares with you how happy she is to see Keely speaking so confidently about her work. In the beginning of the year, Keely’s confidence took a nosedive because she was used to catching on to things quickly. The problems in this class threw her for a loop because they took so long to figure out. Some even stumped her right up until the end of class. This made her feel like she wasn’t smart at math anymore, and she was even considering dropping to a different math class. The teacher had helped the whole class to understand growth mindset and the value of the struggle, and she had worked individually with Keely to help her feel more confident. She said that she viewed this “coaching” aspect of her job to be as important as the teaching aspect.

7. A loud voice in a group in the corner announces, “We don’t have to do all the squares because there’s a pattern and we can use exponents!” The speaker is obviously very excited about his discovery. The teacher walks to the group, probably to explain that they might want to contain their excitement within the group so that everyone else can enjoy the thrill of discovery.

8. A student has left his group with the red cup in his hand, and he is waving it at the teacher’s desk. You and the teacher walk to the desk, and you learn that the student had trouble with last night’s homework on the Law of Sines. The teacher takes a few minutes to lead the student to discover the answers to his questions.

9. You sit down at an empty desk and observe more of the same. The students are very focused and get up only to get more paper or to drop another marble in the jar. The teacher goes from red

cup to red cup to address questions and to check in with students about the meaning and implications of their solutions. When there are no red cups, the teacher listens closely to individual groups discussing the problems, which probably gives her a lot of feedback about what the students understand and how they think. Sometimes she interrupts them to ask questions. You're struck by how much time she spends listening instead of talking.

You notice several interesting things about the students. First, nobody seems to stay frustrated. Any momentary frustration is quickly replaced by renewed determination and new strategies. Second, students are not critical of each other, and nobody is trying to outdo their group members. Third, like in the first group you observed, students are able to point out each other's errors in a non-critical way so that disagreement leads to thoughtful and intelligent discussion.

10. You see a couple of students drop papers into a basket by the window, and you stop one of them to ask about it. He points to a series of 12 posters near the ceiling that say "POTW #13" through "POTW #24." The student explains that these are extra challenges and that the letters stand for Problem on the Wall. The students earn extra points by completing organized solutions and submitting them in the basket. Most of the posters have 6–10 smiley face stickers, which the student explains are added to a poster when someone solves the problem. "Why doesn't #22 have any stickers?" you ask. "The teacher says she hasn't even figured out that one yet. Good luck if you try it." He smiles and you copy the problem down for later.

11. The teacher calls for an intermission and puts $\dfrac{2}{3+\sin\theta}=0.6$ on the board and asks the groups to solve it. While they are working, she explains to you that most of the groups are starting Challenge 3. She knows that they will encounter an equation similar to the one on the board in one of the Challenge 3 problems. She checks groups' answers as they finish, but she never tells them they are going to need this in the Challenges. She explains that she does not want to connect the dots for them, and she says that she uses micro lessons of different types frequently.

12. Most of the groups go back to work, but one of the groups has logged onto a laptop at the computer table. The teacher goes to see what they're doing. Dale, who has taken a leadership role for the group, explains that they have a new idea for the Squirrel Problem. They decided to apply a normal distribution to the randomized variables in the problem, and they want a chance to check it out. The teacher agrees that they can take 8 minutes for this purpose. She knows that this particular group will be able to stay on schedule just fine. You're not able to follow what they're talking about, and you are pretty sure that normal distribution is not part of the geometry curriculum. The teacher explains more to you when she is done talking to the group.

    You learn that the students have been working on the Squirrel Problem for a month. They spend a little bit of class time on it each week, but they also work on it at home. They have to calculate the probability of a squirrel making it across the road safely if they cross the road under a moving car. It's part of a squirrel rite of passage. That's all the information the teacher gave them. They had to figure out how to model the problem and choose the variables that were important. Dale and his group all taught themselves Python so that they could write a program to simulate tens of thousands of squirrel crossings with randomized variables. Several days prior, Dale realized that it would make more sense to randomize the variables using normal distributions about central values. Unfortunately, he had never been taught the mathematics of normal distributions, so he taught himself and his group members so they could apply it to their model and add it to the Python code.

You are obviously dumbfounded that in response to a problem solving prompt, this group learned a programming language and Dale taught himself about normal distributions. It reminds you of the line from *Field of Dreams*, "If you build it, they will come." You are thinking of asking yourself back to class in May so that you can see what amazing things Keely and the other students have done with their independent investigations.

13. You can tell by the rustling of backpacks that the class is almost over. The teacher reminds everyone to finish "AG.05.FC-01 Many Paws & the Law of Sines" so that they can discuss it as a class tomorrow. She tells you that although it's not officially part of the curriculum, the students will need the Law of Sines for one of the Challenges they'll encounter in "Vortex." She explains to you that the assignment will lead the students in developing the Law of Sines, which she believes will bring them to a stronger understanding than if he wrote the formula on the board and had them copy it into their notes and then practice it 20 times.

14. On the way out of the classroom, Felix hands the teacher a solution packet to "AG.05.PWS-05 Putting It in Perspective." You ask if it's a late assignment, and she responds, "No, an extra assignment." The previous week the class had done "AG.05.PWS-03 Flattened Images," which was about finding the coordinates of orthogonal projections of polygons. Felix had chosen to do the follow-up assignment, which the teacher explained was much more difficult because it took into account three dimensions and perspective. The teacher hands you Felix's work. When you look through it, you are very impressed with how organized and methodical it is. You would guess that it came from a mechanical engineering student. "Have others done this extra assignment?" you ask. The teacher tells you that as usual Felix is the first, but she expects that there will be two or three others. "Everyone who does it gets to add five more marbles to the jar, but I think the real reward is the feeling of satisfaction they get from trying it."

15. You thank the teacher, and you tell her that you wish your high school math classes had been like this one. She says that she feels the same way, which is why she created this class the way she did. "I want to show you something." She leads you over to her desk where she opens a bottom drawer and pulls out a shoebox half-filled with letters and cards and sets it on the desk. "These are from former students thanking me for teaching them to think."

